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M/G/`1 Queue With Balking And Generalized Vacations

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***Abstract:** A class of M/G/1 queueing models with a server who is unavailable for occasional intervals of time, independently with the arrival and service processes, can be treated as exceptional queueing system in which the customers who initiate the busy periods have to wait for a random time. In this paper we present a single server queueing system with a finite waiting room and the customer balk from the system.*

Key Words: : occasional, intervals, independently, exceptional, random time, queueing system, finite waiting.

This paper considers a class of M/G/1 queueing models with a server who is unavailable over occasional intervals of time we call a time interval when the server is either unavailable or idle a vacation.

The standard M/G/1 assumptions are applicable here. We consider an M/G/1 system with a finite waiting room is full, the mean arrival rate changes from λ to λ_p (p<1) implying 100 (1-p)% of the arrival balk.

The service in non pre-emptive. That is once selected for service, a customer is served to completion in a continuous manner. Also a vacation model has a property of exhaustive service in case each time that the server becomes available he works in a continuous manner until the system becomes empty.

Before proceeding further, we shall study the following vacation models.

Model I : In this M/G/1 queueing model each time when the system becomes empty, the server begins a vacation of random length. Upon termination his vacation period, the server returns to the system and begins to serve those customers, if any, that have arrived during that vacation. If upon returning, the server finds no customer, he stays in the system waiting for the first one to arrive. This type of models were studied by Levy and Yechiali (1975). Shanthi Kumar (1980) and Minh (1988) with some limitations.

Model II : The assumptions are the same as in model I. If the server returns from a vacation to find no customers waiting, he begins another vacation immediately. The length of vacations are independent of the arrival process. This model has been studied by Miller, Cooper (1970), Levy and Yechiali, Heyman (1977), Scholl and Kleinrock (1983) defined on the M/G/1 queue with rest periods and certain service – independent queueing disciplines.

Model III : The N-policy model – In this M/G/1 queueing model, each time that the system becomes empty, the server waits until exactly N customers are waiting (N is some fixed positive integer), then works continuously until the system is again empty (exhaustive service). It is noting that in this model, the length of a vacation depends on the arrival process. This model was studied by Yadin and Naor (1963) and Heyman (1968) has shown that this N-policy is optimal.

The first two models are generally called queues with vacations. All three belong to a class of models which Fuhrmann and Cooper (1985) called the M/G/1 queue with generalized vacations.

We shall show that, by considering these models as special cases of M/G/1 queueing model in which the customer who initiates a busy period has to wait for a random period, numerous transient results can be obtained.

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We demonstrate in this paper the carry the single server queueing system with balking. We assume that the customer are served in an order that is independent of their service times.

Cooper (1970) obtained queues served in cyclic order, waiting times,, Levy and Yechiali (1975) found utilization of idle time in an M/G/1 queueing system and Minh (1980) analysed the exceptional queueing system by the use of regenerative processes and analytical methods.

We extend the work of Fuhrmann and Cooper (1985) further and present transient results for the waiting times of the M/G/1 queues.

AN M/G/1 QUEUE WITH BALKING :

Let us consider that service does not begin until there are at least r_1 customer in the queue and then r_2 customers or the whole queue which ever is less, are taken into service.

Let

B(x) - service time density function

m - size of finite waiting room.

consider first the case r_1=r_2=1, m=0 using the imbedded Markov chain method we define the transition probabiltes

$$P_{ij} = \text{Pr}ob\left\{n+1 = j \mid n = i\right\} \; n = 1,2,...$$

Then

$$P_{ij} = \begin{cases} k_{j-i+1} & (j \ge i-1,\ i = 1,2,...) \\ k_j & (j \ge 0, \quad i = 0) \\ 0 & (j < i-1,\ i = 2,3,...) \end{cases}$$

where

$$K_j = \int_0^{\infty} e^{-\lambda px}\left\{(\lambda Px)^j / j!\right\} Bx \;\; dx \quad (j = 0,1,2,...)$$

is the probability that j customers join the queue during a service time.

BUSY PERIOD ANALYSIS :

let

X = length of the busy period

G(x) = CDF of the busy period x

let us consider a queueing model in which the customer who initiates the k-th busy period has to wait for a random time y_k before receiving service

$$G^*(s) = \int_0^{\infty} e^{-sx}\, dG(x)$$

be the LST of G(x)

changing the order of integration gives

$$G^*(s) = \int_0^{\infty} \sum_{n=0}^{\infty} \left[\left\{e^{-\lambda t(\lambda t)^n}\right\}/n!\right] dB(t) \int_t^{\infty} e^{-sx} G^n(x-t)\, dt$$

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And by convolution property

$$G^*(s) = \int_0^\infty e^{-\lambda t} e^{\lambda t G^*(s)} e^{-st}\, dB(t)$$

$$= B^*[s + \lambda - \lambda G^*(s)]$$

MAXIMAL WAITING TIME

let

c = initial occupation time of the server

$\eta(t)$ = virtual waiting time at time t

let us define

$$F(x, y \mid c) = \text{Prob}\{\theta_0 \le x \text{ and } n(t) \le y \text{ for } 0 \le t \le \theta_0 \mid n(\theta) = c\}$$

$$(t \ge 0, 0 \le c \le y)$$

be the joint probability distribution of the length of the initial busy period and the maximal virtual waiting time i.e. the time a customer would have to wait if he arrived at time t. We define the LST

$$\phi(s, y \mid c) = \int_0^\infty e^{-sx} dx\, F(x, y \mid c)$$

$$\text{Re}(s) \ge 0, 0 \le c \le y$$

Then we have

$$\phi(s, y \mid c) = w(s, y - c / w(s, y))$$

where

$$\int e^{-sy} dy\, W(s, y) = w / [(w - s - \lambda)](1 - \Psi(w))$$

$$(\text{Re}(w) > \text{Re}[s + \lambda - \lambda g(s)])$$

QUEUES WITH GENERALIZED VACATIONS

Model I :

Let the vacations periods V have a general distribution $V(x)$ with LST $V(z)$. Also let $v = E\{V\}$.

The system can be treated as an exceptional queueing system in which the first customer in the ith busy period has to wait or random time y_i which is equal to the residual life of the server's single vacation period. With

$$\Pr\{y_i = 0\} = \alpha = \int_{x=0}^\infty e^{-\lambda t} dv(t) = v(\lambda)$$

We have

$$Y(z) = G(z) + \alpha = [\lambda V(z) - z\alpha] / (\lambda - z)$$

Where

$$G(z) = \int_{x=0}^\infty \int_{y=0}^\infty \lambda e^{-\lambda y} \exp\{-z(x - y) dy\, dv(x)\}$$

$$= \lambda[v(z) - \alpha] / (\lambda - z)$$

To compare with existing results, we first obtain from the above equation that

$$\bar{Y} = \bar{V} - [1 - \alpha] / \lambda$$

$$E\{n_1\} = [\lambda \bar{V} + \alpha] / [1 - p]$$

Where

$$G(z)=\int_{x=0}^{\infty}\int_{y=0}^{\infty}\lambda e^{-\lambda y}\exp\{-z(x-y)dy\,dv(x)\}$$

$$=\lambda[v(z)-\alpha]/(\lambda-z)$$

To compare with existing results, we first obtain from the above equation that

$$\bar{Y}=\bar{V}-[1-\alpha]/\lambda$$

$$E\{n_1\}=[\lambda\bar{V}+\alpha]/[1-p]$$

Which is equivalent to (6) in Levy and Yachali (1975). Also, from (8) we obtain

$$w(z)=[\lambda-\lambda v(z)+z\alpha]/z[\lambda\bar{V}+\alpha]$$

From this equation we find the equation (22) in Levy and Yachaili (1975).

Model II :

In this system, the probability that a customer arrives at an empty system during the server's K-th (K ≥ 2) vacation period after a busy period, thus initiates a busy period but has to wait for the residual life of this vacation period is α^{K-1}. The LST of the distribution of the waiting times of the customers who initiate the busy periods is therefore

$$Y(z)=G(z)[1+\alpha+\alpha^2+...]$$

$$=\lambda[V(z)-\alpha]/(\lambda-z)(1-\alpha)$$

Again to compare with existing results, we first note that

$$\bar{Y}=\bar{V}/[1-\alpha]-1/\lambda$$

$$E\{n_1\}=\lambda\bar{V}/(1-\alpha)(1-p)$$

which is equivalent to (26) in Levy and Yachiali, This gives

$$W(z)=[1-V(z)]/z\bar{V}$$

which is the LST of the residual life of the vacation periods (see Levy and Yachiali (1975). For this equation, we obtain equation (36) in Levy and Yachiali (1975) which was first derived by Cooper (1970, equation (18))

Model III :

The N-policy model :

(i) With setup time

Let us assume that after any absent period, the server needs a set-up time U before can actually serve customers again. Let

U(z) = E[exp{(-zu)}] for Re (z) ……. 0, U = E{U} , U^2 = E{U^2}

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This model, the regenerative cycle consists of three periods, 'build up' is the first period which starts when the i-th customer arrives to find the system empty. This period terminates when the number of customers in the system is build up to N, i.e. upon arrival of the (i+N)th customer. The second period is the busy period, in which the server is either being set-up or serving a customer. The third period is the empty period in which there is no customer in the system.

System is regenerative therefore amongst the n_1 customers served during the first busy period, there are (N-1) customers arriving during the built-up period. The typical j-th $(1 \le j \le N-1)$ customers arriving during this period has to wait for a duration equals to $(t_j + ... + t_{N-1} + U + S_1 + ... + S_{j-1})$ The N-th customer, who initiates the first busy period has to wait for a duration equal to $Y(z) = U(z)\, S^{N-1}(z)$. This duration is independent with the development of the system often the beginning of the busy period. We therefore set

$$Y(z) = U(z)\, S^{N-1}(z)$$

(ii) With close-down times.

We now assume further that, after serving the last customer in a busy period, before going on a vocation, the server needs a period D to close down the facility. If a customer arrives during this period then the server immediately serve this customer.

That is, if the server cannot complete his closing down attempt, then the next regenerative cycle, starting at the time the arriving customer finds the server closing down, is just a normal busy cycle of the M/G/1 queue, With

$$\theta = \int_0^\infty e^{-\lambda t} dD(t) = D(\lambda)$$

being the probability that the server can complete his close-down process, i.e. is no arrival during the close-down period.

CONCLUSIONS:

We have demonstrated that under certain assumptions the waiting times can be easily adopted to study the above queueing systems. We define three vacation models in this study and also we can derive the transient solutions for queueing systems. Although the results presented here are for the M/G/1 queues only, the method applies equally well for the GI/G/1 queues.

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